Role of OR Techniques in Financial Decision Making

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Abstract
Financial decisions are made aiming at maximum profit with minimum risk factor. Operations Research (OR) techniques play a very important role in analyzing the finance problems such as equity, debt, foreign exchange markets, design securities, market regulations, risk evaluation and control, regulation of capital reserves, devise pricing equations and analyzing market data. These problems have complicated interactions between components or involve a large set of components or alternatives. OR techniques are equipped to formulate such problems as mathematical problems and provide with a feasible solution to such problems. Finance problems are numerical, with well-defined boundaries and objectives, having clear and stable relationship between variables and support availability of accurate data, thus suitable for the application of OR techniques. Financial problems usually involve large sums of money, thus even a small improvement in the quality of solution is beneficial. Programming Techniques of the types like Linear, Quadratic, Non-Linear, Integer, Goal, and Dynamic Programming are mostly used. Among these techniques, Monte Carlo Simulation is most widely used. Some other OR techniques like Network models, Markov Chains and Game Theory are also proposed but less commonly used for the purpose. Whereas otherwise important OR techniques like Queueing Theory and PERT-CPM have not been applied to financial market so far. This study aims at analyzing the role of OR techniques to financial decision making with a special focus on some common types of decision problems in financial markets and the suitable OR techniques which can be used for them.

Keywords
Operations Research, Mathematical Programming, Financial Decisions, Portfolio Selection

I. Introduction
Operations Research, OR, has been applied to problems in finance for at least the last half century. Studies reveal that there is a two way relationship between the techniques of OR and Finance. One, the techniques of OR have been applied to finance problems and OR has influenced financial markets to adopt new finance theories. Two, Finance theories have motivated the development and improvement of OR solution techniques. Finance problems are generally separable and well defined [12]. The objective is usually to maximize profit or minimize risk, and the relevant variables are amenable to quantification, almost always in monetary terms. In finance problems, the relationships between the variables are usually well defined. Thus the resulting OR model is a good representation of reality, particularly as the role of non-quantitative factors is often small. Finance problems also have the advantage that any solution produced by the analysis can probably be implemented, while in other areas there may be unspecified restrictions concerned with human behavior and preferences that prevent the implementation of some solutions. Further more, finance practitioners are accustomed to the quantitative analysis of problems. Also, such problems tend to recur, possibly many times per day, spreading the costs of developing an OR solution over a large number of transactions. This scale and repetition makes the development of an OR model more attractive than for small or one-off decisions.

Finance Problems involve a huge data, large number of components or alternatives and have complicated interactions between components. These problems, after basic assumptions are made, can be formulated as mathematical problems and therefore can be handled with the use of Operations Research. In the presence of new data sources, operational research software tool, intermediaries and emerging data-exchange standards, the use of OR techniques is expected to increase many folds in coming times.

II. Some Real Life Examples
This section has the mention of some very successful continuous applications of OR.

Hewlett-Packard Company uses real option modeling for dynamic pricing and for managing supply/demand risk as a participant in High-Tech Exchange, a procurement-oriented electronic market for companies in computing and related industries.

Market Switch uses constrained optimization to make ad- and promotion placement decisions for Internet advertising networks and e-commerce sites. Its software takes into account the ad promotion pool, associated advertising and partner contracts, and other constraints, working in real time as customers interact with Web sites.

OptiBid, an integer-programming-based, multi-attribute, Internet-enabled, combinatorial auction system, is used by dozens of large shippers to select transportation providers in an online marketplace.

Strategic Data Corp. uses segmentation based on hierarchical clustering, a rule discovery data miner and a real-time learning engine to personalize web-site content for its clients.

Trajecta, Inc. uses predictive and stochastic optimization models to help banks manage their credit-card portfolios, including online applications.

United Sugars Corporation uses web based architecture to deploy its production, distribution, and inventory-capacity optimization application.

III. History
In the earlier study by Zanakis et al. (1986), Management Science or Operations Research was used most frequently [9] in the application areas of financial management, portfolio management, customer credit scoring, and check operations. The most frequently used techniques were statistical analysis, linear programming, forecasting and simulation. More research was needed in productivity/profitability operations and international activities (arbitrage and currency swaps). Data compilation showed compelling evidence that banking priorities have shifted from corporate planning for financial/liquidity management to bank operations, technological advances brought forth sophisticated information systems to aid bankers in daily banking operations and decision-making (DeFarrari and Palmer, 2001), banks have enjoyed greater productivity and performance with the use of MS tools such as Data Envelope Analysis (DEA). However, there are...
areas that still deserve attention.

Table 1: Frequency of Use of MS/OR Techniques’ Applications in Banking

<table>
<thead>
<tr>
<th>Technique</th>
<th>Frequency of Use</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Programming</td>
<td>97</td>
<td>20.17%</td>
</tr>
<tr>
<td>Goal Programming</td>
<td>4</td>
<td>0.83%</td>
</tr>
<tr>
<td>Integer Programming</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>Dynamic Programming</td>
<td>14</td>
<td>2.91%</td>
</tr>
<tr>
<td>Stochastic Programming</td>
<td>45</td>
<td>9.36%</td>
</tr>
<tr>
<td>Forecasting</td>
<td>15</td>
<td>3.12%</td>
</tr>
<tr>
<td>Simulation</td>
<td>25</td>
<td>5.20%</td>
</tr>
<tr>
<td>Queuing</td>
<td>3</td>
<td>0.62%</td>
</tr>
<tr>
<td>Heuristics</td>
<td>4</td>
<td>0.83%</td>
</tr>
<tr>
<td>Statistical Analysis</td>
<td>85</td>
<td>17.67%</td>
</tr>
<tr>
<td>MIS/EDP</td>
<td>18</td>
<td>3.74%</td>
</tr>
<tr>
<td>Other Techniques</td>
<td>171</td>
<td>35.55%</td>
</tr>
</tbody>
</table>

Study found that the preferred techniques used 20 years ago, Statistical Analysis and Linear Programming, are still the top methods in use today. Linear Programming has enjoyed increased popularity due to technological advances such as data envelope analysis (DEA). In their study, Kantor and Mahtar (1999: 30) contended that DEA is used extensively to provide quantitative measures of each branch’s efficiency relative to other similar branches. The most frequently used techniques (as shown in Table 1) are: linear programming, statistical analysis, other methods, stochastic programming, simulation, and MIS/EDP. User familiarity with certain techniques may play a key role in their choice of the techniques.

IV. Some Common Problems in Finance

Finance problems are numerical, with well-defined boundaries and objectives, having clear and stable relationship between variables and support availability of accurate data, thus suitable for the application of OR techniques. Financial problems usually involve large sums of money, thus even a small improvement in the quality of solution is beneficial. So, improvement in quality of solution using techniques is of huge importance. In this section, some important decision strategies are discussed with their handling using techniques from OR.

V. Index-Tracking Fund

Index tracking is a method of passive portfolio management. Using this approach, fund managers attempt to match the performance of a theoretical portfolio when they do not feel confident about the market performance. This matching of the performance of an index can be done in two ways, full replication or partial replication. In full replication, an investment is made in every constituent of the index, proportional to its market share, for example, the issue of a new set of shares. In partial replication, an investment is made in a small proportion of the shares, while attempting to match the performance of the entire index. Partial replication incurs lower initial transaction costs, and is easier to rebalance but has a Tracking Error, the measure of the deviation of the chosen portfolio from the index. The aim of fund managers is to minimize this tracking error. This can be expressed as a quadratic programming problem. The overall approach is that given a subset of constituent shares of the index, the tracking error is the expected squared deviation of return from that of the index. The proportion of capital to be invested in each company is calculated as part of the same problem. This requires two pieces of data, the covariance matrix defining the relationship between all of the companies in the index, and the capitalization weights. The covariance matrix shows the correlations that exist between companies in the index. A large positive value occurs when the return from two companies follow very similar trajectories. A small negative value occurs when two companies follow roughly opposing trajectories. The capitalization weights are simply the normalized returns for the index. They are calculated by taking the return for each company, and dividing it by the sum of the returns for all companies.

Quadratic programming is a minimization procedure for quadratic functions [3], those containing only quadratic terms, linear terms, and a constant. For a given a set of variables, the function can be encoded as a matrix containing the coefficients of the quadratic terms, a vector containing the coefficients of all the linear terms, and the constant. The variables defined in the equations below refer, then, to these vectors and matrices. The function which needs to be minimized in the index tracking problem is given below. The symbols are defined as follows

\[ X - \text{the vector containing the proportion of capital to be invested in each member of the index.} \]

\[ H - \text{the vector containing the capitalization weight of each member of the index.} \]

\[ G - \text{the two dimensional matrix defining the correlations between members of the index.} \]

In their study, Kantor and Maital (1999: 30) contended that DEA is used extensively to provide quantitative measures of each branch’s efficiency relative to other similar branches. The most frequently used techniques (as shown in Table 1) are: linear programming, statistical analysis, other methods, stochastic programming, simulation, and MIS/EDP. User familiarity with certain techniques may play a key role in their choice of the techniques.

The first index-tracking fund was introduced in July 1971[2] by management science group at Wells Fargo Bank in San Francisco.

VI. Portfolio Selection

An important problem in modern finance is the optimal allocation of investment resources among a set of candidate assets. Portfolio selection is the process of selecting on the investment options by an individual or an investment company. The problem of portfolio selection is not very old [5]. In 1952, Harry Markowitz, set the foundations [6] of modern portfolio theory by giving Markowitz portfolio theory. From then on, many authors studied the problem of portfolio selection in many directions.

VII. Mathematical Programming

According to Modern portfolio theory, portfolio selection can be considered as a mathematical problem [6]. Consider a portfolio with \( n \) different assets \( i = 1,2,3,\ldots,n \) with returns \( R_i \). Let \( \mu \) and \( \sigma^2 \) be corresponding mean and variance respectively. \( \Sigma \) be the covariance between \( R_i \) and \( R_j \). Assume the relative amount of value of portfolio invested in asset \( i \) is \( x_i \) and \( R \) is return of whole portfolio then \( \mu = \sum_{i=1}^{n} \mu_i x_i \) and \( \sigma^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j \) such that; such that \( \sum_{i=1}^{n} x_i = 1 \); \( x_i \geq 0 \), \( i=1,2,\ldots,n \)  

Condition \( (*) \) is the same as saying that only long positions are allowed, hence if short sale should be included in the model this condition should be omitted. For different choices of \( x_1, x_2, \ldots, x_n \).
the investor will get different combinations of \( \mu \) and \( \sigma^2 \). The set of all possible \((\mu,\sigma^2)\) combinations is called the attainable set. Those \((\mu, \sigma^2)\) with minimum \( \sigma^2 \) for a given \( \mu \) or more and maximum \( \mu \) for a given \( \sigma^2 \) or less are called the efficient set (or efficient frontier). Since an investor wants a high profit and a small risk he/she wants to maximize \( \mu \) and minimize \( \sigma^2 \) and therefore he/she should choose a portfolio which gives a \((\mu,\sigma^2)\) combination in the efficient set.

### VIII. Bond Refunding Decision

Most corporate bonds contain a provision which allows the issuer to call the bonds before maturity upon repayment of principal plus a premium. This provision raises a series of problems for the financial manager. First given that the firm has outstanding callable bonds should it call them today and if so with bonds of what maturity should it replace them. Second, what is the cost of following an optimum refunding policy over time? While there are a multitude of factors affecting the optimum timing and value of a call, the factor which has been singled out for special attention is the interest savings that can accrue to the firm through the execution of a call. When a firm refunds a bond, it incurs a fixed charge equal to the call premium on the old bond plus the cost of floating a new bond. The refunding itself results in a change in future interest payments equal to the difference in interest payments between the old and the new bonds.

### IX. Example

Assume that

1. Management will maintain a constant level of debt in its capital structure for 5 years after which it will have no debt. This will be referred to as a 5 year time horizon.
2. Management will only issue callable bonds in $100 denominations and with a maturity of 3 years.
3. Management is indifferent on the timing of flows.
4. Floatation expenses on new debt involve a fixed charge of $2.00.
5. The cost of calling the old debt is $2 if it is one year old and $1 if it is two years old. This applies at each decision point as well as at the horizon.
6. The firm currently has a 3 year bond outstanding with a coupon rate of 5% and one year remaining to maturity.
7. Management is willing to base its decision on the point estimates of future interest rates shown in Table 1.

#### Table 1:

<table>
<thead>
<tr>
<th>Time</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

This problem of solving the refunding decision can be handled via dynamic programming. Recursive optimization in a backward direction can be employed. This simple example can be solved using a one equation system. The one equation system is conceptually more difficult but computationally much simpler. Also, it can be used to solve the more realistic problems. We reformulate the problem that the firm floats a bond period ‘t’ when should it refund again. If the firm knew this for every period but the current period it could then calculate the optimum time to refund a bond of any given age in the current period. The optimum time to refund a brand new bond acquired in period four is of course at the horizon when it must be refunded. The cost involved is $2 in floatation to acquire the new bond, $2 in call penalty at the horizon plus the $7 interest charge or:

\[
\begin{align*}
    f_4 &= S + f_7 \quad (\text{Period 4 at horizon})
\end{align*}
\]

The optimum time to refund a brand new bond acquired in period three is either at period four or the horizon. If the firm calls at the horizon it incurs a cost of $2 to float the new bond at period three, $1 in call penalty and 2 interest payments of $7 each. On the other hand, if it calls in period four it incurs $2 to float the new bond, $2 in call penalty, 1 interest payment of $7, and the cost of the optimum policy from period four to the horizon. The firm should choose the least expensive so:

\[
\begin{align*}
    f_3 &= \min \{ F + C(2) + 2R_3 = 2 + 1 + 14 = \$17 \\
    &\quad (t + C(1) + R_4 + f_4 = 2 + 2 + 7 + 11 = \$22 \}
\end{align*}
\]

Other calculations are given in Table 2.

#### Table 2:

<table>
<thead>
<tr>
<th>( t )</th>
<th>( f_t )</th>
<th>( k = 1 )</th>
<th>( k = 2 )</th>
<th>( k = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>11</td>
<td>2 + 2 + 7 + 0 = 11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>17</td>
<td>2 + 2 + 7 + 11 = 22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>2 + 2 + 5 + 17 = 26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>2 + 2 + 5 + 17 = 26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>34</td>
<td>2 + 2 + 4 + 26 = 34</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
    f_{\text{decision}} &= \min \{ 1 + 28 = 29 \\
    &\quad 5 + 26 = 31 \} = 29
\end{align*}
\]

Thus a rather simplistic example is solved by dynamic programming and the bond refunding problems of more realistic formulation can also be solved by dynamic programming.

### X. Conclusion

Mathematical programming of the types - linear, quadratic, nonlinear, integer, goal, chance constrained, stochastic, fractional, DEA and dynamic have been used to solve a considerable range of problems in financial markets. Monte Carlo simulation is also widely used in financial markets is most widely used. In some cases, the use of OR techniques has influenced the way financial markets function since they permit traders to make better decisions in less time. Game theory, decision trees, Queuing theory and Inventory models are less used for handling finance problems. OR techniques can also be used by financial regulators and financial institutions in setting capital adequacy standards. OR techniques play an important role in financial decision making and, with the recent dramatic improvements in the real time availability of data and in computer speed, this role will increase. This will create the opportunity for OR techniques to play an even greater role in financial decision making.

### References


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