Abstract
The Indian Stock Market is made up of participants from tick traders to long-term investors. Each participant having different investment horizons trade simultaneously in the market. This paper finds evidences of long memory in the returns from the data collected from Indian Bombay Stock Exchange. Fractal Market Hypothesis (FMH) explores the application of chaos theory and fractals to finance. The main idea of the fractal model is that the investors may not directly react to information. Instead, the investors may normally react with delay. The standard complement of statistical techniques used to identify predictable market structure, is capable of identifying regular periodic cycles. The Indian stock market has a high degree of persistence.

Keywords
Persistent, Long Range, Dependence, Rescaled Range Analysis, Hurst Exponent, Non-Periodic cycles, Bombay Stock Exchange

I. Introduction
The nonparametric statistical techniques provide a viable alternative testing for nonlinear structure and are ideal for modelling financial data as they make prior assumption about the distribution of data. One such nonparametric technique is Rescaled Range Analysis (Hurst H.E -1951). The Rescaled Range value is called the rescaling range. In a time series, the range increases with increments of time. Rescaled Range Analysis is a useful tool for revealing some well disguised properties of stochastic time series such as persistence and anti-persistence characterized by non-periodic cycles (Jonathan M Blackledge-2008).

The Fractal Model accounts for markets that consist of many irrational investors, trading at different investment horizons. As long as the market maintains a fractal structure with no characteristic time scale, the market should remain stable. If all investors’ time horizon were to become unique, the market would become unstable because everyone would respond according to the same information at the same time. The standard complement of statistical techniques used to identify predictable market structure is capable of identifying regular periodic cycles (McKenzie Michael D-2001).

II. Literature Reviews
The previous studies analyzed the long memory and market cycles in stock markets are reviewed below.

III. Statement of the Problem
A stock market crash is often defined as a sharp dip in share prices of equities listed on the stock exchanges. In parallel with various economic factors, a reason for stock market crashes is also due to panic and investing public’s loss of confidence. Often, the stock market crashes end speculative economic bubbles. The new information regarding securities comes to the market in a random fashion and the timing of one announcement is generally independent of others. The competing investors attempt to adjust the security prices rapidly to reflect the effect of new information. If all investors’ time horizon were to become unique, the market would become unstable because everyone would respond according to the same information.
It is to be noted that there was the lack of quantitative evidences about the potential bias of equity returns in Indian Stock Market and the failure of standard statistical analysis to uncover any long-run trends or cycles in financial market data. As of today, there is no empirical evidence that supports or rejects the random walk model. Hence an attempt is made to study the long range dependence in the Indian Stock Market.

IV. Need of the Study
This study using the Rescaled Range Analysis, a useful tool for revealing stock price returns of stochastic time series such as persistence and long range dependence characteristics by non-periodic cycles. The purpose of R/S analysis is to provide an assessment of how the apparent variability of a series changes with the length of the time-period being considered and to determine the fractal dimension of a time series. In assessing the fractal dimension of asset returns, the investors may better understand the systematic pattern of price returns and consequently may adjust their asset pricing strategies accordingly.

V. Objectives of the Study
The objective of the study is to analyze the stationary and to examine the long range dependence of monthly index returns of Sensex.

VI. Hypotheses of the Study
The following null hypotheses are tested.

NH1 - The monthly index returns of Sensex are non stationary and
NH2 - The monthly index returns of Sensex have no long range dependence.

VII. Methodology of the Study

A. Sources of Data and Computational Details
The monthly index returns of BSE Sensex were used to examine the evidence of non-periodic cycles and long range dependence. BSE is the oldest stock exchange in Asia. The equity market capitalization of the companies listed on the BSE was US$1 trillion as of December 2011, making it the 6th largest stock exchange in Asia and the 14th largest in the world. The BSE has the largest number of listed companies in the world. The required data were collected from the PROWESS, a corporate database. The other required data were collected from Books, Journals and various Websites.

B. Period of the Study
The study period covered a period of ten years from 1st January, 2002 to 31st December, 2011.

C. Tools Used in this Study
The following tools were used for this study.

1. Descriptive Statistics
The descriptive statistics include Mean, Median and Standard Deviation.

2. Augmented Dickey Fuller Test
In statistics and econometrics, an Augmented Dickey–Fuller test (ADF) is a test for a unit root in a time series sample. The more negative it is, the stronger the rejection of the hypothesis that there is a unit roots at some level of confidence. The testing procedure for the ADF test is the same as for the Dickey–Fuller test but it is applied to the model

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \cdots + \delta_p \Delta y_{t-p} + \varepsilon_t,$$

where $\alpha$ is a constant, $\beta$ the coefficient on a time trend and $p$ the lag order of the autoregressive process. By including lags of the order $p$ the ADF formulation allows for higher-order autoregressive processes. This means that the lag length $p$ has to be determined when applying the test. The unit root test is carried out under the null hypothesis $\gamma = 0$ against the alternative hypothesis of $\gamma < 0$.

3. Rescaled Range Analysis
Hurst (1965) developed the rescaled range analysis, a statistical method to analyze long records of natural phenomena. There are two factors used in this analysis: firstly the range $R$, which is the difference between the minimum and maximum ‘accumulated’ values or cumulative sum of $X(t,\tau)$ of the natural phenomenon at discrete integer-valued time $\tau$ over a time span $\tau$, and secondly the standard deviation $S$, which is estimated from the observed values $X(t)$. Hurst found that the ratio $R/S$ is very well described for a large number of natural phenomena by the following empirical relation

$$\frac{R}{S} = (C \cdot T)^H$$

where $\tau$ is the time span, and $H$ the Hurst exponent. The coefficient $c$ was taken equal to 0.5 by Hurst. $R$ and $S$ are defined as:

$$R(\tau) = \max_{1 \leq \tau \leq T} \sum_{i=1}^{\tau} X(t, \tau) - \min_{1 \leq \tau \leq T} \sum_{i=1}^{\tau} X(t, \tau)$$

and

$$S = \left( \frac{1}{T} \sum_{\tau=1}^{T} \left( \sum_{i=1}^{\tau} \{ X(t) - \bar{X} \}^2 \right) \right)^{1/2}$$

Where, $\bar{X} = \frac{1}{T} \sum_{t=1}^{T} X(t)$ and

$$X(t, \tau) = \sum_{i=1}^{\tau} \{ X(i) - \bar{X} \}$$

VIII. Limitations of the Study
The following are the major limitations of the study
1. The study was confined only to the Sensex returns.
2. As the study was based mainly on secondary data, it was beset with certain limitations which were bound to arise while dealing exclusively with secondary data, and
3. All limitations associated with various tools like Descriptive Statistics, Augmented Dickey Fuller Test and Rescaled Range Analysis, are also applicable to this study.

IX. Analysis of Stationary and Long Range Dependence in the BSE Sensex Returns
For the purpose of this study, the analysis is made as follows
1. Analysis of Descriptive Statistics of BSE Sensex returns
2. Analysis of Augmented Dickey-Fuller test (ADF) of BSE Sensex returns
3. Estimating the Hurst Exponent using Rescaled Range Analysis of BSE Sensex returns
4. Estimating the Hurst Exponent for non-periodic cycles of BSE Sensex returns, and
5. R/S Chart for 20 months, 30 months, 50 months, 80 months and 120 months period of BSE Sensex returns
A. Analysis of Descriptive Statistics of BSE Sensex Returns

Table-1 reveals the results of Descriptive Statistics for BSE Sensex Returns from January 2002 to December 2011. The perusal of the above Table clearly shows that Sensex had earned positive average returns of 0.016464 and it implies that the price series have slowly increased over the period of time. The Sensex returns earned the standard deviation of 0.0768 which reveals the risk in the market. The negative skewness i.e., -2.06832 indicates a distribution with an asymmetrical extending towards more negative values. It is clearly understood from the analysis that the market was negative skewness characterized the degree of asymmetry of a distribution around its mean. The Kurtosis measure of positive return distribution was characterized the relative peakedness, whose value was 4.150352. Thus the possibility of a distribution was leptokurtic as the value of kurtosis was positive. It indicates that the investors can mean a greater chance of extremely negative outcomes in BSE during the study period.

B. Analysis of Augmented Dickey-Fuller (ADF) of BSE Sensex

The results of ADF Test with regard to the BSE Sensex returns from January 2002 to December 2011 are presented in Table-2. According to above Table, the statistical value for the Augmented Dickey-Fuller (ADF) was -9.557375 and its probability was 0.0000. The test critical values were -3.486064 at 1% level, -2.885863 at 5% level and -2.579818 at 10% level. It is to be noted that the calculated statistical value (-9.557375) was lesser than the critical values (-3.486064) at 1% level, (-2.885863) at 5% level and at 10% level (-2.579818). It is clear that the ADF Test provided the evidence of stationary in respect of monthly Sensex returns. The overall analysis of ADF test proved that the null hypothesis (NH1) - The monthly index returns are non stationary, is rejected. It was not necessary to analyse the data at the further level as it was stationary at the unit root test itself.

C. Estimating the Hurst Exponent using Rescaled Range Analysis on BSE Sensex

It is to be noted that if Hurst Exponent value H is close to 0.5, it indicates a random walk of the return. If Hurst Exponent value H lies between 0 and 0.5, it means that time series exhibits “anti-persistent behaviour”. It indicates that the prices increasing in the past imply the prices decreasing in the future and vice versa. If Hurst Exponent value H lies between 0.5 and 1, it means that the time series has “persistent behaviour”. It implies the fact that the prices increasing trend in the past clearly implies the increasing trend in the future prices too and vice versa.

The monthly index returns movements for 50 months are presented in Figure-2. The above Figure clearly reveals the persistence of time series. Thus the investors may share the same risk level with different investment horizon, which would make the market stable.

D. Estimating the Hurst Exponent for Non-Periodic Cycles on BSE Sensex

An added advantage of rescaled range analysis lay in its ability to discern cycles within data. This not only includes regular periodic cycles but also non-periodic cycles as well. Table-4 shows the Hurst Exponent values from January 2002 to December 2011. The analysis of Hurst Exponent for Non Periodic Cycles of Sensex reveals the Hurst Exponent value in the long range dependence. It is to be noted that none of the values for the time lags was equal to 0.50, indicating that the Indian Stock Market did not follow random walk in so far as the monthly returns were concerned. This shows that there was a definite possibility for persistence in the Sensex returns but the values for time lags for 80 months were close to 0.50, indicating that the trend was not perfectly established. However, for the longer periods, the values were reasonably higher than 0.5, indicating a definite possibility for persistence. Therefore, the investors may share the same risk level with different investment horizon, which would make the market stable.

E. R/S Chart for 20 Month, 30 Month, 50 Month, 80 Month and 120 Month Period of BSE Sensex Returns

The movements of monthly index returns of Sensex using Rescaled Range Analysis were given in figs. 1 to 5. Fig. 1 shows the monthly index returns of Sensex for 20 months. The plots of log(R/S) and log(N) represents the empirical rescaled ranges of the data set for 20 months. It is to be noted that the log (R/S) values slowly increases while the log (N) period (i.e., the returns of Sensex values) increasing during the period of 20 months. The above Figure clearly reveals the persistence of time series. Therefore the movement of stock market (BSE) was predictable during the study period. The monthly index returns of Sensex movements for 30 months are depicted in Figure-2. The above Figure reveals that the price trend moves upward direction during the period of 30 months. It shows the downward direction of price trend during the beginning of a study period, but not for the entire period of 30 months. Therefore the analysis of Hurst exponent value for 30 months detects the persistence in time series. Thus the investors may easily determine the price distribution of a time series. The monthly index returns movements for 50 months are presented in Figure-3. According to the above Figure, the series of plots move towards the upward direction through the period of 50 months. It implies that the series of data clearly reveals persistence, as the time series reflect the increasing trend. Thus the current market prices impact the future price trend forever. This may be a result of longer returns periods picking up trends in the stock prices in the BSE Sensex returns.

The plots, as shown in the above Figure, clearly indicate that the trend was not perfectly established. Because, the plots shows upward and slightly downward trend for the period of 80 months, as the Hurst value is nearly 0.50 (i.e.,0.5271). However, for the longer period, the series were reasonably increasing through the period of 80 months, indicating a definite possibility for persistence of Sensex returns. It reveals the fluctuations in Sensex returns of 80 months covering more distance as the tendency of the price to continue its current direction. Hence the investors can easily predict the stock prices in the market during the study period.

The monthly index returns movements for 120 months. The stock market was persistence, as the price trend moves...
the upward direction throughout the period. It demonstrates that stock market participants habitually overreact to past news and information. Thus, the series of data reveals the long memory of Sensex returns, when the numbers of observations are increasing.

X. Conclusion and Discussion

The present study examined the long range dependence in the BSE Sensex returns using the Rescaled Range Analysis. It was found that there was persistence in stock market during 20 months, 30 months, 50 months, 80 months and 120 months as all measures of Hurst range from 0.5 to 1. It indicates the index returns of Sensex confirmed persistence during the study period. It is clear that the investors reacted to the information being received from past news, reveals high degree of persistence in BSE Sensex returns. Besides, the rescaled range analysis confirmed persistent behaviour of Sensex returns.

Edgar E. Peters (1989) tested the behaviour of stock and bond returns of Standard & Poor 500 and suggested that the prices are reflected on the basis of stronger influence of investors’ sentiment. Muller U.A, et.al., (1995) examined the foreign exchange prices and found that the time value and fractal properties lead to Heterogeneous market. The other studies by Hsing Fang et.al., (1994), John T Barkolas et.al., (1999) and Gayathri M and Selvam M (2011 b) also confirmed the time series having fractal dynamics characterised by long term dependence and non periodic or irregular cycles in the market by using the Rescaled Range Analysis. A similar study was conducted by Assaf .A (2006) who investigated the presence of fractional dynamics in the returns and volatility of the emerging stock market of Kuwait. The Variance Rescaled statistic for the returns which indicated that there was persistence and long range dependence in the Emerging Stock market of Kuwait. The findings of Hsing Fang et.al., John T Barkolas et.al., Gayathri M and Selvam M and Assaf .A confirmed the findings of the present study.

Murugesan Selvam et.al., (2011) by using fractal structure analysis in Indian Stock Market found that the market trends occasionally followed the random walk. The findings of present study did not confirm the findings of Murugesan Selvam et.al., Thus the study described the fractal structure of a series at long lags. There is persistent temporal dependence even between distant observations as a series exhibited long memory. Hence, the identification of persistent and long range dependence in the market may give rise to profitable trading opportunities to the investors and technical analysts.

References


Table 1: Descriptive Statistics for Sensex Returns from 2002 to 2011

<table>
<thead>
<tr>
<th>Descriptive Variable</th>
<th>Sensex</th>
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<tbody>
<tr>
<td>Mean</td>
<td>0.0164</td>
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<tr>
<td>Maximum</td>
<td>0.2826</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.2389</td>
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<tr>
<td>Standard Deviation</td>
<td>0.0768</td>
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<tr>
<td>Skewness</td>
<td>-0.206832</td>
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<tr>
<td>Kurtosis</td>
<td>4.150352</td>
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<tr>
<td>Number of observations</td>
<td>120</td>
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</table>

Source: Computed from PROWESS Corporate Database Using E-Views 5.1
Table 2: Augmented Dickey-Fuller Test for Sensex Returns from 2002 to 2011

<table>
<thead>
<tr>
<th>t-Statistic</th>
<th>Probability</th>
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<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-9.557375</td>
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<tr>
<td>Test critical values:</td>
<td></td>
</tr>
<tr>
<td>1% level</td>
<td>-3.486064</td>
</tr>
<tr>
<td>5% level</td>
<td>-2.885863</td>
</tr>
<tr>
<td>10% level</td>
<td>-2.579818</td>
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</tbody>
</table>

Source: Computed from PROWESS Corporate Database Using E-VIEWS 5.1

Table 3: Fractal Dimension for Sensex Returns from 2002 to 2011

<table>
<thead>
<tr>
<th>Statistic Value</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Rescaled range analysis</td>
<td>0.5690</td>
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<tr>
<td>Standard Deviation</td>
<td>0.0768</td>
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<tr>
<td>Fractal dimension</td>
<td>1.4289</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>120</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
<td>2</td>
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</tbody>
</table>

Source: Computed from PROWESS Corporate Database Using KaotiXL 1.1 Software

Table 4: Hurst Exponent for Non Periodic Cycles of Sensex from 2002 to 2011

<table>
<thead>
<tr>
<th>Period N</th>
<th>20 months</th>
<th>30 months</th>
<th>50 months</th>
<th>80 months</th>
<th>120 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hurst Exponent</td>
<td>0.5457</td>
<td>0.7393</td>
<td>0.5710</td>
<td>0.5271</td>
<td>0.5711</td>
</tr>
</tbody>
</table>

Source: Computed from PROWESS Corporate Database Using KaotiXL 1.1 Software

Fig. 1: Chart Showing R/S for 20 Months

Fig. 2: Chart Showing R/S for 30 Months

Fig. 3: Chart Showing R/S for 50 Months

Fig. 4: Chart Showing R/S for 80 Months

Fig. 5: Chart Showing R/S for 120 Months
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