Solving Transportation Problem with Mixed Constraints

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Abstract
A transportation problem basically deals with the problem, which aims to find the best way to fulfill the demand of n demand points using the capacities of m supply points. Here we studied a new method for solving transportation problems with mixed constraints and described the algorithm to find an optimal More-For-Less (MFL) solution. The optimal MFL solution procedure is illustrated with numerical example and also computer programming. Though maximum transportation problems in real life have mixed constraints, these problems are not be solved by using general method. The proposed method builds on the initial solution of the transportation problem which is very simple, easy to understand and apply.

Keywords
Transportation Problem, Mixed Constraints, More-For-Less Paradox, Modified VAM Method

I. Introduction

One of the most important and successful application of quantities analysis to solving business problems has been in the physical distribution of products, commonly referred to as Transportation Problems (TP). Basically, the purpose is to minimize the cost of shipping goods from one location to another so that the needs of each arrival area are met and every shipping location operates within its capacity. The TP finds application in industry, planning, communication network, scheduling, transportation and allotment etc. In real life, however, most of the problems have mixed constraints but we used TPs for optimal solutions with equality constraints. The TPs with mixed constraints are not addressed in the literature because of the rigor required to solve these problems optimally. A literature search revealed no systematic method for finding an optimal solution for TPs with mixed constraints.

The More-For-Less (MFL) paradox in a TP occurs when it is possible to ship more 'total goods' for less (or equal) 'total cost' while shipping the same amount or more from each origin and to each destination, keeping all shipping costs non-negative. The occurrence of MFL in distribution problems is observed in nature. The mixed constraints TP have extensively been studied by many researchers in the past years [5,8-9]. Gupta et al. [7] and Arsham [6] obtained the more-for-less solution, for the TPs with mixed constraints by relaxing the constraints and introducing new slack variables. While yielding the best more-for-less solution, their method is very hard to understand since it introduces more variables and requires solving sets of complex equations. Later Adlakha et al. [1- 4] developed a heuristic algorithm for solving TP with mixed constraints, which is based on the theory of shadow price. In the previous year Pandian et al [10-13] also studied new method to solve TP with mixed constraints.

In this paper, we introduce a modified VAM method for solving TPs with mixed constraints in MEL paradoxical situation. The optimal MFL solution procedure is illustrated with the help of numerical example and computer programming. The proposed method is very simple, easy to understand and apply. The MFL situation exists in real life and it could present managers with an opportunity for shipping more units for less or the same cost.

II. Formulation of Transportation Problem with Mixed Constraints

Let m be the number of sources and n be the number of destinations. Suppose that the cost of transporting one unit of the commodity from source i to the destination j is c_{ij}. Let a_i be the quantity of the commodity available at source i and b_j be the quantity required at destination j. Thus $a_j \geq 0$ and $b_j \geq 0$ for all i and j. Then the general formulation of the transportation problem with mixed constraints, as described by Pandian and Natarajan [10], is

Table 1:

	1	2	3	... ...	n	Supply
1	c_{11}	c_{12}	c_{13}	... ...	c_{1n}	$\leq/=/\geq a_1$
2	c_{21}	c_{22}	c_{23}	... ...	c_{2n}	$\leq/=/\geq a_2$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	c_{m1}	c_{m2}	c_{m3}	... ...	c_{mn}	$\leq/=/\geq a_m$
Demand	$\leq/=/\geq b_1$	$\leq/=/\geq b_2$	$\leq/=/\geq b_3$	... ...	$\leq/=/\geq b_n$	

If x_{ij} is the quantity transported from source i to destination j then the transportation problem is written with the help of Adlakha et al. [3] and Pandian and Natarajan [13] as

$$Minimize\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$Subject\ to\ \sum_{j=1}^{n} x_{ij} \leq / = / \geq a_i, i = 1, 2, \ldots\ldots\ldots\ldots m$$

$$\sum_{i=1}^{m} x_{ij} \leq / = / \geq b_j,\ j = 1, 2, \ldots\ldots\ldots\ldots n$$

$$x_{ij} \geq 0$$

The above formulation represents a Linear Programming Problem (LPP) with $m \times n$ variables and $m + n$ constraints. If the LPP is small, we can solve the problem by using any simplex method, but in practical life LPP can be very large, which is difficult to solve by analytically. This type of problem can be solved very easily by using computer programming.

Remark 1: If all constraints are of equal (=) sign, then the problem becomes the transportation problem with equality constraints.

III. Proposed Method

We propose the following algorithm based on VAM method for finding an optimal solution to a transportation problem with mixed constraints.

Step 1: For each row and column of the transportation table, find the difference between the two lowest unit shipping costs. These numbers represent the difference between the distribution cost on the best route in the row or column and the second best route in the row or column.

Step 2: Identify the row or column with the greatest opportunity cost or difference.

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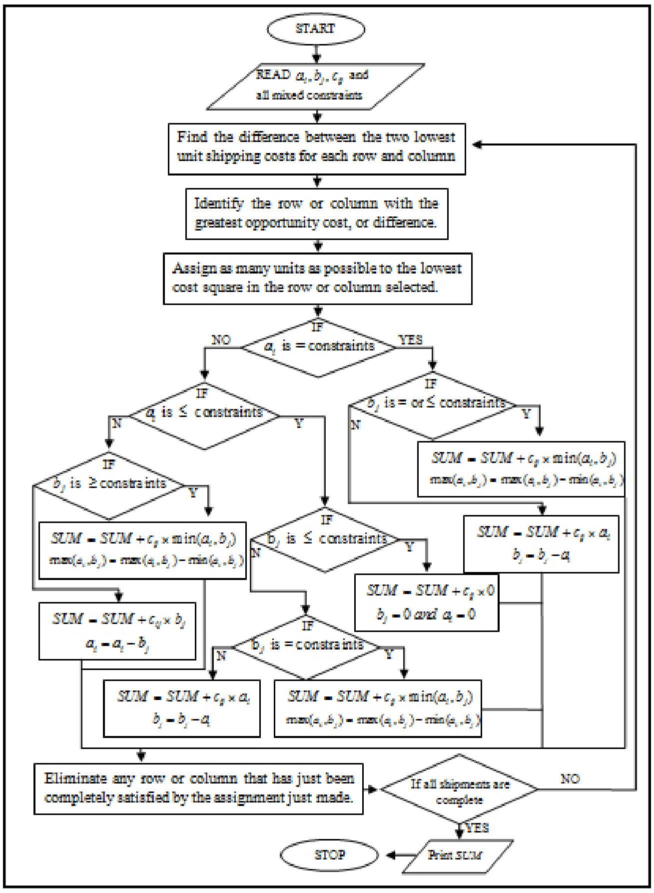
Step 3: Assign as many units as possible to the lowest cost square in the row or column selected. (If the assignment unit contains $\leq$ $\leq$ sign, then assign as lowest unit as possible. If the assignment unit is of $\geq$ sign, then assign the possible maximum value.) We will follow the following chart to assign the supply and demand unit.

IV. Flow Chart



We will follow the following chart to assign the supply and demand unit.

Step 4: Eliminate any row or column that has just been completely satisfied by the assignment just made.
Step 5: Recomputed the cost difference for the transportation table.
Step 6: Return to step 2 and repeat the steps until an initial feasible solution has been obtained.

Table: 2

Chart		
supply a_i	demand b_j	assign unit
=	=	$\min(a_i, b_j)$
=	≤	$\min(a_i, b_j)$
=	≥	a_i
≤	≤	0
≤	=	$\min(a_i, b_j)$
≤	≥	a_i
≥	≥	$\min(a_i, b_j)$
≥	=	b_j
≥	≤	b_j

V. Numerical Examples

We explain the proposed method for finding an optimal solution to a transportation problem with mixed constraints. First we transformed the problem into LPP, then solve it by using simplex method and computer programming. Finally, we solve the problem by the proposed method and verify the result.

A. Example 1

Table 3:

	1	2	3	*Supply*
1	2	5	4	= 5
2	6	3	1	≥ 6
3	8	9	2	≤ 9
Demand	= 8	≥ 10	≤ 5	

Now transformed this problem into LPP as
Minimize

$$Z = 2x_{11} + 5x_{12} + 4x_{13} + 6x_{21} + 3x_{22} + 1x_{23} + 8x_{31} + 9x_{32} + 2x_{33}$$

Subject to,

$$x_{11} + x_{12} + x_{13} = 5$$
$$x_{21} + x_{22} + x_{23} \geq 6$$
$$x_{31} + x_{32} + x_{33} \leq 9$$
$$x_{11} + x_{21} + x_{31} = 8$$
$$x_{12} + x_{22} + x_{32} \geq 10$$
$$x_{13} + x_{23} + x_{33} \leq 5$$
$$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33} \geq 0$$

We solved this problem both by hand calculation and computer programming. By the hand calculation using simplex method, after eleventh iteration we find that the minimum transportation cost is 58 unit. By the computer programming on simplex method, developed by ourselves, we get the same result as shown bellow.

Output:

X_{11} = 5.000000
X_{12} = 0.000000
X_{13} = 0.000000
X_{21} = 3.000000
X_{22} = 10.000000
X_{23} = 0.000000
X_{31} = 0.000000
X_{32} = 0.000000
X_{33} = 0.000000

Minimum of Objective Function = 58.00000

It should be noted here that, Pandian and Natarajan [12] used the Fourier method for solving this problem and obtained the same result.

Now, we solve the above problem by using our proposed method. First, we solve the problem by hand calculation and then by using computer programming. Hand calculation solution is shown in Tables 4-6.

Table 4:

	1	2	3	*Supply*
1	2	5	4	$= 5[2]$
2	6	3	1	$\geq 6[2]$
3	8	9	2 (0)	$\leq 9[6] \leftarrow$
Demand	$= 8$ [4]	≥ 10 [2]	≤ 5 [1]	

In Table 4, we first find the difference between the two lowest unit shipping costs in each row and column and find that row 3 has the largest difference as shown in the table by $\leftarrow$ mark. We find that 2 is the lowest cost square in row 3. Now we assign as many units as possible in C_{33}. Since demand and supply are of both '$\leq$' sign, so according to the chart provided the assignment unit is 0. Now we recomputed the cost difference in the same way and proceed in the next step until we get the feasible solution.

Table 5:

	1	2	3	*Supply*
1	2 (5)	5	**4**	$= 5[3]$
2	6	3	**1**	$\geq 6[3]$
3	**8**	**9**	**2** (0)	$\leq$ **9**
Demand	$= 8/3$ [4] $\uparrow$	≥ 10 [2]	$\leq$ **5**	

In Table 5, we find that 2 is the lowest cost square in column 1. Now we assign as many units as possible in C_{11}. We see that the demand and supply are both '$=$' sign, so using the chart we get our assignment unit as 5. We combine all the working in one table and shown in Table 6:

Table 6:

	1	2	3	*Supply*
1	**2** (5)	5	4	$= 5[2][3]$
2	**6** (3)	**3** (10)	1	$\geq 6[2][3]$
3	8	9	**2** (0)	$\leq 9[6]$
Demand	$= 8/3$ [4] [4]	≥ 10 [2] [2]	≤ 5 [1]	

In the cost matrix C_{33} we see that the supply and demand are both '$\leq$' sign, so we assign the lowest possible value in C_{33} In C_{11} the supply unite is 5 so we cannot supply more than 5 units. In C_{22} the supply unit is ≥ 6, so we can supply more than 6 units. Therefore, the solution for the given problem is $x_{11} = 5, x_{21} = 3, x_{22} = 10$, and all other $x_{ij} = 0$ for a flow of 18 units with the total transportation cost as 58, which is equal our previous result.

By the computer program depending on our proposed method, we get the result as shown bellow.

TRANSPORTATION PROBLEM

3.000000	3.000000	0.0000000
2.000000	2.000000	30.00000
2.000000	1.000000	18.00000
1.000000	1.000000	10.00000

MINIMUM TRANSPORT COST IS: 58.0

Thus we see that the solution obtained by simplex method and by our proposed method is the same.

B. Example 2

The X Clothing Group owns factories in three towns that distribute to four dress shops (A, B, C). Factory availabilities, projected store demands and unit shipping costs are summarized in the table below:

Table 7:

	A	B	C	D	Factory Availability
1	12	4	9	5	$= 55$
2	8	1	6	6	≥ 40
3	1	2	4	7	≤ 30
Store Demand	$= 40$	$= 20$	≤ 45	≤ 20	

Now we transformed the above problem into LPP as

Minimize

$$Z = 12x_{11} + 4x_{12} + 9x_{13} + 5x_{14} + 8x_{21} + 1x_{22} + 6x_{23} + 6x_{24} + 1x_{31} + 2x_{32} + 4x_{33} + 7x_{34}$$

Subject to,

$$x_{11} + x_{12} + x_{13} + x_{14} = 55$$

$$x_{11} + x_{12} + x_{13} + x_{14} = 55$$

$$x_{21} + x_{22} + x_{23} + x_{24} \geq 40$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 30$$

$$x_{11} + x_{21} + x_{31} = 40$$

$$x_{12} + x_{22} + x_{32} = 20$$

$$x_{13} + x_{23} + x_{33} \leq 45$$

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$$x_{14} + x_{24} + x_{34} \le 20$$
$$x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34} \ge 0$$

We solve the above problem by hand calculation and after eleventh iteration we find that the minimum transportation cost is 605 unit. By the computer programming, we get the following result.

Output:

X_{11} = 0.000000
X_{12} = 0.000000
X_{13} = 35.00000
X_{14} = 20.00000
X_{21} = 10.00000
X_{22} = 20.00000
X_{23} = 10.00000
X_{24} = 0.000000
X_{31} = 30.00000
X_{32} = 0.000000
X_{33} = 0.000000
X_{34} = 0.000000

Minimum transportation cost = 605.0

We find that our hand calculation result and computer oriented solution is the same. Now we solve the above problem by using our proposed method both by hand calculation and computer programming solution. Hand calculation solution is shown in Tables 8-10.

Table 8:

	A	B	C	D	F.A.
1	12	4	9	5	$= 55$ [1]
2	8	1	6	6	≥ 40 [5]
3	1 (30)	2	4	7	≤ 30 [1]
S.D.	$= 40$ [7] ↑	$= 20$ [1]	≤ 45 [2]	≤ 20 [1]	

Table 9:

	A	B	C	D	F.A.
1	12	4	9	5	$= 55$ [1]
2	8	1 (20)	6	6	≥ 40 [5] ←
3	**1 (30)**	**2**	**4**	**7**	**≤ 30**
S.D.	$= 40$ [4]	$= 20$ [3]	≤ 45 [3]	≤ 20 [1]	

In Table 8, first we find the difference between the two lowest unit shipping costs in each row and each column and identify that column 1 has the greatest difference as shown in the table with ↑ mark. Here 1 is the lowest cost square in the column 1, now we assign as many units as possible in C_{31}. We see that the demand is of '$=$'sign and supply is of '$\le$' sign in first column and third row respectively, so from the chart as provided earlier, we find that the assignment unit is 30 (min (a_i, b_j)). Now we recomputed the cost difference in the same way as shown in Table 9, for solving the next step.

We show all workings combining in one table as shown in Table 10:

Table 10:

	A	B	C	D	F.A.
1	12	4	9 (35)	5 (20)	$= 55$
2	8 (10)	1 (20)	6 (10)	6	≥ 40
3	1 (30)	2	4	7	≤ 30
S.D.	$= 40$	$= 20$	≤ 45	≤ 20	

From Table 10, the solution of the given transportation problem with mixed constraints is:

$$x_{13}=35, x_{14}=20, x_{21}=10, x_{22}=20, x_{23}=10, x_{31}=30$$

and all other $x_{ij} = 0$ for a flow of 125 units with the minimum transportation cost as 605 unit, which is equal to our previous result. By the computer programming of our proposed method, we get the following result:

TRANSPORTATION PROBLEM

1.000000 2.000000 80.00000
1.000000 4.000000 100.0000
1.000000 3.000000 135.0000
2.000000 3.000000 180.0000
2.000000 1.000000 80.00000
3.000000 1.000000 30.00000

MINIMUM TRANSPORT COST IS: 605.0

Thus we see that the solution obtained by simplex method and by our proposed method is the same.

VI. Conclusion

We have provided a modified VAM algorithm to find a solution for the transportation problems with mixed constraints. At first we transformed the problem into LPP and then solved it by using simplex method. We also developed computer program for solving such problems by simplex algorithm. We then developed a new method for solving transportation problem of More-For-Less (MFL) solution with mixed constraints and wrote computer program for solving such problem, and verify that our computer program is correct. Thus our newly developed method with computer program saves time and energy and easy to apply.

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